EXTENDED VARIATIONAL PRINCIPLES FOR FIC EQUATIONS

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The Finite Increment Calculus (FIC) develops modified and stabilized forms of the classical field equations of mechanics by introducing characteristic lengths as coefficients of residual derivative terms. For the finite element discretization of FIC equations, weighted residuals and Petrov-Galerkin methods have been favored.

For problems that are essentially conservative, for example elastic solid mechanics, use of Petrov-Galerkin methods seem unnecessary and may delay consideration of FIC methods by FEM practicioners already trained in energy formulations. Thus it is appealing to place FIC within a variational framework. The use of a classical variational framework runs into difficulties because of the presence of odd-derivative terms in the FIC equations. However, extended variational forms, which require the introduction of modified field variables, may be used to construct functionals that recover the FIC equations as Euler-Lagrange equations.

In this article extended variational principles are derived for the equations of solid mechanics with and without first derivative terms. The discrete finite element equations are applied to one-dimensional benchmark problems. When first derivative terms are missing, as in the classical problem of a bar, the optimal characteristic length for nodally exact solutions is zero, in which case one recovers the standard FEM equations. But for benchmark problems containing both first and second derivative terms, nonzero optimal steplengths emerge.